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1993 J. Phys. A: Math. Gen. 26 2037

(http://iopscience.iop.org/0305-4470/26/8/024)

J. Phys. A: Math. Gen. 26 (1993) 2037–2046. Printed in the UK

QED between parallel mirrors: light signals faster than c, or amplified by the vacuum

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Received 29 September 1992

Abstract. Because it is scattered by the zero-point oscillations of the quantized fields, light of frequency ω travelling normally to two parallel mirrors experiences the vacuum between them as a dispersive medium with refractive index $n(\omega)$. Our earlier low-frequency result that $n(0) < 1$ is combined with the Kramers–Kronig dispersion relation for n and with the classic Sommerfeld–Brillouin argument to show (under certain physically reasonable assumptions) that either $n(\infty) < 1$, in which case the signal velocity $c/n(\infty)$ exceeds c; or that the imaginary part of n is negative at least for some ranges of frequency, in which case the vacuum between the mirrors fails to respond to a light probe like a normal passive medium. Further, the optical theorem suggests that n exhibits no dispersion to order e^4, i.e. that $n(\infty) = n(0)$ up to corrections of order e^6 at most.

1. Introduction and conclusions

Consider the Maxwell field at absolute zero temperature in the region between two plane-parallel mirrors a distance L apart. We idealize by taking them to be indefinitely extended, and perfectly conducting at all frequencies however high; these idealizations define an instructive model, and here we shall not question them further. The boundary conditions at the mirrors ($E_{\parallel} = 0 = B_{\perp}$) constrain the normal modes of the field, and when the field is quantized, its vacuum structure differs from that in unbounded space. Different in particular are the vacuum (zero-point) expectation values of the squared field components and of the energy density; the latter is lowered, as witnessed by the Casimir effect (for a recent review see, e.g., Mostepanenko and Trunov 1988).

It is well known too that (even in the absence of mirrors) the zero-point motions of the electron–positron (Dirac) field profoundly alter the properties of the vacuum in (fully interacting) QED relative to those in classical physics: for instance, they induce nonlinearities in Maxwell's equations and a consequent scattering of light by light. These nonlinearities, jointly with the mirror-induced changes in the zero-point Maxwell field, then cause the speed of light between (and normal to) the mirrors to differ from and possibly to exceed c. Though the differences are too small by many orders of magnitude ever to be observed in practice, we think that they raise interesting matters of principle, and study them in this paper without undue diffidence.

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We stress that such a speed greater than c between mirrors does not in any way contradict or pose any conceptual threats to special relativity, though admittedly it can prove eye-catching because at first sight one might think that it does. The presence of the mirrors breaks Lorentz invariance along the mirror normal (the mirrors define a preferred inertial frame), which obviates the arguments used in special relativity to prove that no signals can travel faster than light does in unbounded (Lorentz-invariant) space. By contrast, Lorentz invariance is unbroken parallel to the mirrors, and the light speed in these directions naturally must, and does, remain unchanged.

There is no need, at the outset, to ask what if any boundary conditions are to be imposed on the Dirac field at the mirrors. Such conditions can have consequences of two kinds, local and global. Locally, they alter the fermion propagators, but these changes decrease exponentially with distance from the mirrors (on a scale set by the Compton wavelength), and here we shall ignore them altogether. The global effects bear on the selection rules, i.e. on the question whether a sufficiently energetic photon (excitation of a Maxwell normal-mode between the mirrors) can or cannot, to order e^2, decay spontaneously to an electron–positron pair. We shall return to this question only when it becomes acute, in section 5. Meanwhile, readers preferring to have a specific model might like to think of the Dirac field as subject to periodic boundary conditions, which forbid decays of the kind just described.

At frequencies $\omega \ll m$ well below the electron mass m, and for field strengths well below m^2/e†, the nonlinear corrections to Maxwell's equations are summarized by the Euler–Heisenberg effective Lagrangean density (Berestetskii *et al* 1982, sections 127, 129; Barton 1991)

$$\Delta\mathscr{L} = \frac{1}{2^3 3^2 5\pi^2}\frac{e^4}{m^4}\{(\boldsymbol{E}^2 - \boldsymbol{B}^2)^2 + 7(\boldsymbol{E}\cdot\boldsymbol{B})^2\}. \tag{1}$$

$\Delta\mathscr{L}$ is the basis of the only explicit calculations to date (Scharnhorst 1990a, Barton 1990), which show, for propagation normal to the mirrors, that at such non-relativistic frequencies the effective refractive index n of the vacuum between two mirrors is constant and less than 1, whence the phase and group velocities are equal, and greater than c. These results and their limitations are summarized in section 2.

At higher frequencies $\omega \geqslant m$, explicit calculations would have to use the full armoury of relativistic QED, adapted to mirrors as regards the Casimir effect for instance by Bordag *et al* (1985). Regarding the propagation of light such explicit calculations are likely to prove extraordinarily awkward, and none have been done to date. Section 3 spells out some of the difficulties, due mainly to the unreliability of perturbation theory at high frequencies. The present paper aims to sidestep this awkwardness by deriving some conclusions through simple but very general arguments, based on local causality as embodied in the dispersion relation for $n(\omega)$. Section 4 writes down the dispersion relation and discusses its status. If one then assumes in the orthodox way that the imaginary part of n is non-negative, then it follows at once that $n < 1$ at infinite (as well as at zero) frequency; and consequently that the true signal velocity normal to the mirrors, $c/n(\infty)$, exceeds c just as the low-frequency phase and group velocities do. One can escape this conclusion only if $\operatorname{Im} n$ is negative at least for some ranges of frequency, which would mean that the vacuum can amplify a light signal, i.e. that its response to a probe beam is quite unlike that of any normal passive medium.

† We use natural units $\hbar = 1 = c$, and unrationalized Gaussian units for the Maxwell field. Thus the fine-structure constant is $e^2 \approx \frac{1}{137}$.

We believe that in strict logic the choice between these two alternatives remains open; indeed, subjectively the two present writers probably incline to opposite choices. On the one hand, it is a matter of observation that the consequences of $n(\infty)<1$ continue to shock most physicists, however weakly such a reaction might be rooted in physics. On the other hand, we know of no explicitly sustainable models with the alternative property of negative Im n, which is counterindicated in at least two ways. First, section 5 outlines some arguments about likely mechanisms responsible for Im n, which suggest that it vanishes to order e^4. If so, then, up to correction of order e^6 at most, n is the same real constant less than 1 (by an amount of order e^4) at *all* frequencies. (The reason why we do not regard this argument as final proof of Im $n>0$ is just that it is perturbative, and as such may fail at high frequency.) The second counterindication to negative Im n is the sheer perversity of the immediate implication that the vacuum between the mirrors amplifies a weak incident probe beam. Subject to energy conservation, it would then seem that the probe must be triggering a pre-existent instability; the no-photon state in Fock space would turn out to have been a false vacuum; and perturbation theory would fail through its incapacity to describe the subsequent collapse.

2. The low-frequency refractive index: phase and group velocities exceeding c

Between the mirrors, the propagation of (say) a plane-wave probe is modified, with respect to unbounded space, by the fermion-induced coupling of the probe fields (treated as external fields) to the zero-point oscillations of the quantized Maxwell field. A study of the effective Maxwell action between mirrors in the regime $\omega \ll m$ governed by (1) showed (Scharnhorst 1990a) that, to leading order, the operative refractive index for propagation normal to the mirrors becomes

$$n=1+\delta n \qquad \delta n=-\frac{11\pi^2}{2^2 3^4 5^2}\frac{e^4}{(mL)^4}. \tag{2}$$

By contrast, for propagation parallel to the mirrors the refractive index remains unity as in unbounded space, and as dictated by the surviving mirror-compatible Lorentz invariance under boosts in these directions†.

The reduction of n below 1 is related to the reduction of the zero-point energy-density below its value in unbounded space‡, and thereby to the Casimir effect. It should perhaps be stressed that the position-independent shift $\delta n=\frac{1}{2}(\delta\varepsilon+\delta\mu)$ results from shifts $\delta\varepsilon(\delta\mu)$ in the dielectric constant (magnetic susceptibility) that do vary with position. Taken separately they diverge at each mirror, whence equation (2) cannot be applied closer to the mirrors than a Compton wavelength or so. Elsewhere, $\delta\varepsilon$ and $\delta\mu$ vary on a scale of L, whence the local refractive index n suffices to describe light propagation in the geometric-optics (WKB) regime, i.e. with wavelengths well below L (Barton 1990).

† Though derived for ideal mirrors, these refractive indices apply equally with real mirrors that become transparent above some frequency ω_0, provided $\omega_0 L \gg 1$.

‡ By zero-point effects we always mean the appropriate quantity evaluated in the presence of the mirrors, with the corresponding mirrors-absent quantity subtracted. Such renormalization supplies finite and mathematically sensible answers to all physically sensible questions, even for perfect mirrors: i.e. divergences are eliminated without appealing to the transparency of real mirrors at high frequencies. (At least this is the case for plane-parallel geometry, and here we consider no other.)

Equation (2) shows that (as long as $\omega \ll m$) n is frequency-independent and less than 1. Hence the phase and group velocities normal to the mirrors are equal and both greater than 1:

$$\frac{\omega}{k} = \frac{d\omega}{dk} = \frac{1}{n} \approx 1 - \delta n = 1 + |\delta n| \qquad \omega \ll m. \tag{3}$$

As already stressed in section 1, δn is too small to measure in practice. Moreover, on a single traverse the low-frequency prediction (2) cannot be verified even in principle, because any wavegroup narrow enough to afford the requisite accuracy must include significant high-frequency components to which the effective coupling (1) and therefore (2) no longer apply. To see this, note that to determine $\delta c = -\delta n$ directly as a velocity-change one would need a wavepacket of width δx, where $\delta x / L \lessdot \delta c / c \sim |\delta n|$, so that $1/\delta x \gtrdot 1/L|\delta n|$. But the wavenumber width δk of such a packet obeys $\delta k \gtrdot 1/\delta x$, whence it must include components with

$$\frac{k}{m} \gtrdot \frac{1}{L|\delta n| m} \sim \frac{(Lm)^3}{e^4} \gg 1. \tag{4}$$

This has been pointed out by Ben-Menahem (1990). Milonni and Svozil (1990) give an argument to the same effect, but based on the switch-on times and the durations of light signals available from excited atoms.

Clearly, such considerations are not specific to the effect we are studying. What they show, equally for quantum and for classical waves, is that in confined geometries a single measurement on a single traverse can determine the speed of light having limited frequencies only with limited accuracy†: under such conditions the operational significance of any ultimate speed is apt to remain somewhat nebulous. (As so often when applying the indeterminacy relations, one could argue that the statistically analysed average of many measurements does make it possible, in principle, to determine shifts well below the mean-square deviations, and thus to verify effects that more cursory considerations of single measurements sometimes describe as undetectable. However, this is not the place to pursue such very wide questions relating to measurement theory in general.)

3. Signal speed and the high-frequency refractive index

In order to determine the true signal speed, i.e. the speed of a sharp wavefront advancing into an initially undisturbed medium, one needs to consider the propagation of, say, the electric field of such a wavegroup. Careful reasoning initiated by Sommerfeld establishes (se e.g. Brillouin (1960), and for a lucid textbook treatment Jackson (1975)) that this may be written as

$$E(t, z) = \int_{-\infty}^{\infty} d\omega\, a(\omega) \exp\{-i\omega t(1 - n(\omega) z/t)\} \tag{5}$$

† We do not know whether repeated traverses could do the trick. The difficulty is that every extra traverse involves an extra reflection, with the light crossing the narrow region next to the mirror where (2) is unreliable. This in turn might well entail a time delay (or advance) τ, which we cannot calculate. Thus, without some reason to suppose that $|\tau| \ll L/\delta c \sim L|\delta n|$ for practicable L, multiple traverses offer no demonstrated advantage. Alternatively, one might try to construct a measurement scenario on the perhaps plausible assumption that τ (like the singularities of $\delta\varepsilon$ and $\delta\mu$ right next to the mirrors) is independent of L; but such attempts soon become uncomfortably far-fetched.

where $a(\omega)$ is analytic in the upper-half complex ω plane; and that the signal speed† is $1/(\mathrm{Re}\, n(\omega \to \infty))$. But $n(\infty)$ is a quantity not accessible from the low-frequency coupling (1), whose consequence (2) we shall henceforth write as $n(0)$.

Regarding the relevance to wave propagation of $n \equiv \sqrt{\varepsilon\mu}$ at high frequency, we reason as follows. Since the geometric-optics (WKB) approximation (in terms of a local refractive index) suffices to describe the propagation of a probe beam even in the low-frequency regime as soon as $\omega \gg 1/L$, it seems clear on physical grounds that this approximation can only improve as one enters the high-frequency regime $\omega \geqslant m \gg 1/L$. Likewise it seems clear that n will remain independent of position (except within a few Compton wavelengths from the mirrors).

Unfortunately, as far as we know no-one as yet has calculated $n(\infty)$. The situation is the same in some other problems of light-propagation, e.g. in gravitational background fields (Drummond and Hathrell 1980, Dolgov and Khriplovich 1983); in external electromagnetic fields (Bialynicka-Birula and Bialynicki-Birula 1970, Adler 1971, Ternov *et al* (1982), and at finite temperature (Barton 1990, 1991). All these problems have been investigated at roughly the same level of technical and conceptual rigour as ours. What is calculated in every case is only the behaviour of n in the appropriate low-frequency regime; the central technical difficulty too is essentially common to all cases, and we spell it out only for ours. The point is that for $\omega \ll m$ and $mL \gg 1$ one can convince oneself without too much trouble that the two-loop contribution (2) dominates all higher loops; therefore the calculation of $n(0)$ is under proper control, and a credible result can be evaluated to leading (fourth) order in e. By contrast, as $\omega \to \infty$, there is no reason to neglect the contributions of loops of arbitrarily high order. To calculate $n(\infty)$ is therefore a truly non-perturbative task: improvements on (2) by means of perturbation theory cannot yield definitive information about the signal velocity of light between mirrors. Accordingly, in the next section we fall back on simple and general arguments from a few basic physical principles, in preference to speculating about technicalities in a way that (short of a complete calculation) can carry little conviction.

Another point of view has been put by Ben-Menahem (1990), who notes that the signal speed cannot exceed c unless commutators of the Maxwell fields (response functions) can cease to vanish beyond the original unperturbed (mirrors-absent) light cone; and that to arrive at such an outcome through a perturbative calculation, the true QED interaction (between the Maxwell and the Dirac fields) would in effect have to introduce suitable delta-function derivatives into the commutators on that cone‡. Further, he draws a largely verbal sketch of a calculation which his final paragraph then cites as grounds for believing that this cannot happen, and that wavefronts between mirrors consequently do travel exactly at the speed c. However, we have been unable to discern mathematical evidence for his sketch, and will not therefore elaborate technical reservations about ways in which one might conceivably try to formalize it. This does not of course mean that the conclusion is necessarily wrong. In our view, it means only that for the present the question (whether or not $n(\infty)$ is unity) remains open, and that it can be settled only by actually calculating the commutators; this in

† As pointed out by Ben-Menahem (1990), one of us (Barton 1990) failed, earlier, to distinguish properly between group and signal velocity.

‡ From a canonical viewpoint it may be worth noting that there is no proven need for the *equal-time* commutation rules to change even if the signal speed were to exceed c. In that case the effective light-cone is distorted between the mirrors, but 'causality' in the sense commonly associated with the existence of a light cone would continue to apply, provided one makes due kinematic allowance for the distortion.

turn will need demonstrably adequate approximations to the requisite high-frequency dynamics†. If, by whatever means, it were eventually established that $n(\infty)=1$, then, as explained in the introduction, one would be driven to the second alternative, namely to negative Im n, which is discussed further in section 5 below.

4. The alternatives allowed by the Kramers–Kronig relation

Our purpose here is not to debate whether $n(\infty)$ equals unity, but merely to connect $n(\infty)$ with the undisputed result $0<n(0)<1$ through the standard 'Kramers–Kronig' dispersion relation for $n(\omega)$‡, which is the only secure non-perturbative relation available. (It makes no difference whether we work with n or $\varepsilon\mu=n^2$.) This connection will show that one must accept at least one of two equally unorthodox possibilities: either $n(\infty)<1$, so that the true signal velocity, too, exceeds c; or the conventional no-photon vacuum between the (fixed!) mirrors amplifies a probe beam. In the second case the vacuum would fail to act as a passive medium; though such failure is often *associated with instability, we prefer to label this scenario through the lack of passivity*, in order not to prejudice the argument through other meanings that are sometimes associated with 'unstable vacua'. We revert to the physical implications in the last paragraph of this section.

For the real part of n, the dispersion relation reads (see, e.g., Newton 1982)

$$\operatorname{Re} n(\omega)=\operatorname{Re} n(\infty)+\frac{2}{\pi}\int_0^\infty d\omega'\,\frac{\omega' \operatorname{Im} n(\omega')}{\omega'^2-\omega^2} \tag{6}$$

assuming only that n converges at infinity (see point (iv) below); more precisely, Im $n(\omega\to\infty)$ needs to vanish. Before exploiting (6) we add some comments to clarify its status.

(i) The relation stems directly from the analyticity of $n(\omega)$ for Im $\omega>0$; this in turn follows via Titchmarsh's (1948) theorem from local causality, i.e. from the fact, quite unrelated to the speed of light, that at a given point there can be no polarization before there is a polarizing field§.

(ii) A monochromatic plane wave would be proportional to the exponential in the integrand of (5). A passive medium, since it can absorb but not amplify, must have Im $n(\omega)\geqslant 0$ for all ω. Some explicit observations about Im n are made in section 5 below.

† In fact, explicit calculation (Scharnhorst 1990b) shows that any dynamics that ultimately entails $n(\infty)<1$ does also, at the proper stage, automatically deliver the delta-derivatives that Ben-Menahem claims can never occur.

‡ Though the frequency spectrum of the normal modes between and propagating perpendicularly to the mirrors is wholly discrete, this fact does not at all prevent one from defining and (in principle) determining the response (described by $n(\omega)$) of the inter-mirrors vacuum to probes of arbitrary (continuously variable) frequency. The crucial mathematical point is that the probe beams originate from sources (of freely variable frequency), whence they satisfy the inhomogeneous rather than the homogeneous wave equation. For an explicit discussion see Barton (1989).

§ Regarding ordinary media with both spatial and temporal dispersion (i.e. where the wavenumber k is not dictated by the frequency ω), it is quite a delicate matter to decide whether the properly causal response functions (to which the dispersion relations apply) are the $\varepsilon(\omega, k)$ and $\mu(\omega, k)$, or their inverses. (For a good review see Kirzhnitz (1989).) Fortunately our archetypal disperison relation for n (or equivalently for the forward scattering-amplitude of light) is immune to such subtleties.

(iii) For a conventionally granular dilute medium (number density ρ) the Rayleigh construction (see, e.g., Ditchburn 1976) yields

$$n = 1 + 2\pi\rho f/\omega^2. \tag{7}$$

The optical theorem then entails

$$\mathrm{Im}\, n(\omega) = 2\pi\rho\, \mathrm{Im}\, f(\omega)/\omega^2 = \rho\sigma(\omega)/2\omega. \tag{8}$$

Here f is the coherent forward scattering amplitude and σ the total cross section from a single isolated target particle. Heuristically, it is tempting to try to adapt this construction to our problem by identifying the target particles with the zero-point photons between the mirrors (see the second footnote on page 2039). Unfortunately, though at non-relativistic frequencies it works, we have found that attempts to quantify this approach at high frequencies are fraught with unresolved though fascinating difficulties (Barton 1992). Meanwhile, all that we can safely assert is that, in (7), (8), a properly implemented field-theory calculation would replace ρf by F, and $\rho\sigma$ by Σ, where F is the total coherent forward photon scattering amplitude from unit volume between the mirrors, and Σ the corresponding total cross section.

(iv) We have no conclusive proof that Re n converges and Im n vanishes as $\omega \to \infty$, because, as pointed out in section 3, their asymptotics are quite likely to be governed by non-perturbative effects that we cannot calculate. Nevertheless it is very plausible that n does behave in this way, especially in the light of the analogy suggested by the Rayleigh construction (cf comment (iii) above). To bring this to bear, we first need some photon–photon kinematics. Consider the collision, in the mirror-fixed frame, between a photon from the probe beam, travelling in the z-direction (along the mirror normal) with frequency ω, and a zero-point photon with frequency ω_λ and wavevector having polar angle Θ_λ. Denote the photon frequencies in the centre-of-momentum frame by ω'. Then ω' and the usual Mandelstam s-variable read $s = 4\omega'^2 = 2\omega\omega_\lambda(1-\cos\Theta_\lambda)$.

It follows directly from (8) that Im n could remain non-zero only if the total cross section for a photon propagating between (and normally to) the mirrors were to grow at least linearly with ω, which is very hard to imagine. By contrast, the total light–light cross section to order e^4 behaves like $1/s$, while the exact cross section (to all orders) is thought to approach a constant (Berestetskii *et al* 1982, sections 127, 134); if so, then the kinematics above entail Im $n \sim 1/\omega^2$ and Im $n \sim 1/\omega$ respectively. (For comparison, the Froissart bound on two-particle scattering admits only total cross sections growing no faster than $\log^2(s)$.)

In any case, it will become evident that the conclusions in this section remain conveniently immune to non-perturbative effects, as long as these do not cause the dispersion relation (6) to diverge; such immunity follows simply because perturbation theory does suffice to determine $n(0)$. On the other hand, if (6) did diverge, then our conclusions would certainly have to be reconsidered using subtracted dispersion relations (Nussenzveig 1972).

We are now in a position to exhibit our central alternative, simply by evaluating (6) at zero frequency. With $n(0)$ and $n(\infty)$ real, we may write

$$n(\infty) = n(0) - \frac{2}{\pi}\int_0^\infty \frac{\mathrm{d}\omega}{\omega}\, \mathrm{Im}\, n(\omega). \tag{9}$$

From (2) we know that $n(0) < 1$. If the vacuum between the mirrors is passive, then $\mathrm{Im}\, n \geqslant 0$, whence the integral is non-negative, $n(\infty)$ like $n(0)$ is less than 1, and the true signal speed exceeds c. Conversely, if the true signal speed is c, i.e. if $n(\infty)=1$, then Im n must be negative at least in some ranges of frequency, whence the vacuum between the mirrors cannot behave passively at all frequencies.

Between these two possibly uncomfortable alternatives we do not here propose a definitive choice. That the first is perfectly compatible with relativity theory properly understood has already been spelt out in section 1. If nevertheless one wished to avoid it, then Im n would have to be capable of assuming negative values without physically unacceptable implications.

Such a scenario has been proposed for light propagation through gravitational background fields, where the vacuum bears some resemblance to the vacuum between mirrors (Birrell and Davies 1982, Fulling 1989), though the latter is much clearer conceptually and much easier to handle. The gravitational case has been studied by Drummond and Hathrell (1980), who took account of quantum corrections and, long before us, encountered a refractive index $n(0)<1$. Since this situation too lacks Lorentz invariance, there would not, in our view, be anything necessarily wrong about a signal velocity exceeding c. Nevertheless, Dolgov and Khriplovich (1983), rather than entertain this possibility, seek to escape it by observing that even an impeccably orthodox Im n can turn negative, provided the medium is inhomogeneous in a way that can focus the beam (and thus lead to a local increase in amplitude without any global creation of photons). While their observation is undoubtedly correct, it does not apply to our case, where, as explained in section 2, the refractive index is independent of position. Under such conditions Im $n<0$ would signify induced generation of real photons, with the vacuum between the mirrors behaving as an active medium; *prima facie* this would seem to entail energy creation out of nothing, a scenario for which it might prove difficult to devize a physically reasonable interpretation (see e.g. the last sentence of section 1).

5. Absorption and dispersion between mirrors

In considering Im n, which determines n via the Kramers–Kronig relation (6), it is probably best to disregard the weight of implications piled onto the outcome by section 4, and to proceed as straightforwardly as possible from first principles. In such an approach pursued by perturbation theory it is automatic that Im n is governed by the mechanisms available to absorb photons, which cannot make Im n negative.

We start by adopting a version of QED where fermions as well as photons are strictly confined to a slab of width L: then fermion momenta too are quantized in the z-direction. In slab-space QED†, essentially the same kinematic selection rule as in unbounded space prevents a photon from decaying into an electron–positron pair at a rate of order e^2. Further, energy-momentum conservation prevents the decay of a single initial photon into any final state via tree diagrams alone. However, no reason

† The chief advantage of slab-space QED is that it is mathematically well defined and physically clean even though somewhat artificial. If photons are confined to the slab while fermions are not, then the conservation rules allow all sorts of peculiar processes, with peculiar consequences, for which physical interpretations are fanciful at best. For instance, a photon between the mirrors (whose z-momentum is quantized) could then decay spontaneously into an electron-positron pair (whose z-momenta are not quantized), even to order e^2, entailing for instance Im $n\sim e^2(\omega-2m)/Lm^2$ just above threshold, and Im $n\sim e^2/L\omega$ when $\omega\gg m$ (Barton 1992). Here we ignore such oddities, and models that admit them. (The reader will realize that mirrors reflecting photons with $\omega\gtrsim m$ are pretty odd anyway.)

is apparent why a photon with $\omega > 2m$ should not decay spontaneously into a pair via closed-loop mechanisms involving zero-point photons (i.e. photon propagators) which supply momentum but no energy; in other words, the mirrors absorb the recoil. (One might expect that in the main such decays originate close to the mirrors.)

Some elaboration of these arguments leads directly to certain remarkable conclusions about absorption and dispersion as described by $n(\omega)$. The imaginary part of n is proportional to the imaginary part of the forward scattering amplitude F introduced at the end of comment (iii) in section 4. Thence, using the field-theory version of the optical theorem (see, e.g., Itzykson and Zuber 1980), one can convince oneself that Im F and Im n vanish not only to order e^2 as already mentioned, but also to order e^4.

To see this, consider the two 2-loop diagrams contributing to F, shown in the figure; we disregard the 1-loop photon polarization diagram, which is clearly irrelevant. Adapting the Cutkosky rules to slab-space QED, one can cut these diagrams arbitrarily, and thus obtain Im F. The crucial point is that in every case at least one of the two pieces of the cut diagram stands for a kinematically forbidden process, whose amplitude therefore vanishes. Thus, by virtue of the optical theorem, Im F and thereby Im n vanish in slab-space QED to order e^4. The dispersion relation then entails that the real part of the refractive index is constant to order e^4: to this order we have $n(\omega) = n(0) = n(\infty)$†. Absorption and dispersion start only to order e^6.

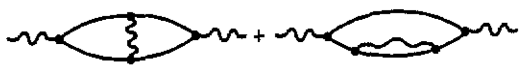

Figure. Two-loop diagrams contributing to F and hence to the refractive index n.

Acknowledgments

KS thanks D Robaschik, M Bordag and E Wieczorek for discussions.

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† Technically speaking, the exact two-loop expression for $n(0)$ yields some function $Q(mL)$, which has not yet been calculated exactly. However, its leading term for $mL \gg 1$ is known and proportional to $1/(mL)^4$; this is the dominant contribution leading to the shift δn in (2).

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